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BOOK OF ABSTRACTS

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Sooryanarayan is a PhD Scholar working under the guidance of Prof. Ramasubramanian at the Cell for Indian Sciences and Technologies in Sanskrit, IIT Bombay. He is also a Upakurvaṇa-brahmacārī and the Director of Education Initiatives at Anaadi Foundation, located near Palani in Tamil Nadu. Prior to being a sadhaka and a research scholar, he was the past C.E.O. and co-founder of EduSeva Technologies.

Nārāyaṇa Paṇḍita's Novel "Third Diagonal" in Cyclic Quadrilaterals

In his comprehensive mathematical treatise *Gaṇitakaumudī* (c.1356 CE), Nārāyaṇa Paṇḍita has presented a nuanced and systematic exposition of cyclic quadrilaterals along with its key properties, through eloquent verses. In his creative mathematical approach, Nārāyaṇa fashions a "third diagonal" by interchanging two sides of a cyclic quadrilateral. The third diagonal is key for Nārāyaṇa Paṇḍita to build upon and provide the relation for various properties such as area, altitude, and circumradius employing the third diagonal in them. This presentation shall bring out the verses of Nārāyaṇa, trace his mathematical ideas, and present modern mathematical derivations for the properties of the third diagonal presented by him.

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Dr. Raghavasimhan Thirunarayanan completed his M.S. and PhD in electrical engineering from EPFL, Lausanne, Switzerland. Currently, he works as a Senior Analog / R.F. Design Engineer in 3dB Technologies, Zurich where he is involved in system and chip design for next-generation keyless access. He also works on Indian mathematics, exploring ancient mathematical techniques and their applicability to modern mathematics. He is currently a scholar-in-residence at IIT Gandhinagar under the HoMI project.

Indian techniques to solve Pell's equations

Pell's equations which are Diophantine equations of the form $x^2 - Dy^2 = 1$ have been a source of immense research over a period of many centuries. Indian mathematicians have contributed greatly to this with their discovery of the *bhāvanā* in the 7th century CE, followed by the *cakravala* process of Jayadeva in the 11th century CE. But there were also a few lesser-known techniques such as the ones espoused by Śrīdhara as well as Jayadeva. In this talk, we briefly give an overview of these techniques as well as provide pointers for their further exploration which might open avenues for new algorithms for solving Pell's equations.

Prasad Jawalgekar

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Prasad Ashok Jawalgekar, having acquired his bachelor's in engineering, took interest in the history of science in India. He is currently pursuing his research toward obtaining a doctoral degree at IIT Bombay.

Applications of the Third Diagonal of Cyclic Quadrilateral

Nārāyaṇa Paṇḍita's mathematical treatise *Gaṇitakaumudī*, by the virtue of being the largest among the extant *pāṭīgaṇita* works, contains in it a gamut of wide-ranging topics and unique sections, approaches, and illustrative examples. It is quite interesting to note that close to one-fourth of the verses in the entire text of *Gaṇitakaumudī* are present in its fourth chapter *Kṣetra-vyavahāra* – devoted to geometry – thus making it the biggest of the fourteen chapters. And within the 243 verses of *Kṣetra-vyavahāra*, Nārāyaṇa Paṇḍita deals with cyclic quadrilaterals and spreads it over one-third of the chapter. With 80+ verses, he has presented a nuanced and systematic exposition of cyclic quadrilaterals along with a detailed discussion of their key properties. In his creative mathematical approach, Nārāyaṇa fashions a "third diagonal" by interchanging two sides of a cyclic quadrilateral. Then he provides the precise mathematical expression for getting the magnitude of the third diagonal in terms of its sides. He also provides precise mathematical relation that enables one to express several components of a cyclic quadrilateral involving the third diagonal. The present study attempts to unearth the applications of this third diagonal in finding the area, altitude, segments of diagonals from their intersection, and circumradius of a cyclic quadrilateral.

Nivitha Rose Thomas & Ardra Prajesh

Stella Maris College, Chennai

Nivitha Rose Thomas completed high school in Kerala. She is currently doing her Undergraduate degree in Mathematics at Stella Maris College, Chennai. She is interested in pursuing her higher degree in Business Administration. A multi-disciplinary approach is her forte. This attitude has kindled her interest in presenting a paper on integrating Mathematics and the Mother of all Indian languages, Sanskrit. She is pursuing her foundation course in Sanskrit under the Part-1 Language program.

Ardra Prajesh is an Undergraduate student pursuing her Bachelor's degree in Mathematics at Stella Maris College, an institution for women's education located in Chennai. Ardra is interested in furthering her studies and wishes to do her Master's in Mathematics. She has chosen Part 1 language, Sanskrit, and completed three exams with good grades. Currently, she is in II years pursuing her UG degree. Her interest includes learning Indian languages and their literature. She has presented projects on the Fourier Series. She is self-motivated and curious to learn subjects like Psychology, Behavioural Science, Statistics etc. Her inquisitive approach to learning ancient knowledge systems has motivated her to probe this topic of integrating Sanskrit & Mathematics.

How Piṅgala Created the Binary Number System

The binary number is the fundamental principle on which today's digital technology works. Piṅgala, the ancient Indian prosodist and mathematician was the first to develop and use the binary number system while studying and analysing Sanskrit poetic meters in his work *Chandaḥśāstra*. The binary number system is a place value system where you have only two digits 0 and 1, which are called bits. In the binary number system, the value of the digits depends on its position ie digit times the power of 2.

Letters in Sanskrit can be either long or short syllables. A verse in classical Sanskrit literature has four quarters or *pada* in a stanza. The metre of a verse is determined by the number of syllables and their order of arrangement in a quarter. A short syllable is one ending with one of the short vowels, which are a (अ), i (इ), u (उ), ṛ (ऋ) and ḷ (ऌ). Even a short syllable will be a guru if what follows is a conjunct consonant, an *anusvara*, or a *visarga*. The long syllable is defined as one with one of the long vowels, which are ā (आ), ī (ई), ū (ऊ), ṝ (ॠ), e (ए), ai (ऐ), o (ओ) and au (औ). The last syllable of a foot of a metre is taken to be guru optionally.

It is interesting to observe that in Sanskrit poetry, the concept of binomial coefficients, Fibonacci numbers and binary numeration has been in use right from the days of Piṅgala who was the first to write a treatise on *Chandaḥśāstra* relating to metres in Sanskrit poetry.

Each quarter may have the same number of syllables or the same number of time units, a short one being assigned 1 time unit and a long one 2-time units. There are metres in which the odd quarters have the same number of syllables or time units, while the even quarters have a different number of equal units.

A Sanskrit stanza or *padya* consists of four *padas* or four quarters, which are regulated by (i) The number of syllables in each quarter, or (ii) The number of syllabic time units or *mātras*, a short sound being assigned one unit of time and the long one. Each stanza or *padya* can thus be of two broad types, *vṛtta* or the one in which metre is regulated by the number and composition of syllables, and the other *jati*, where the metre is regulated by the number of syllabic instants or time units in each quarter 8 sets of triplets of long or short sounds or a mixture of both have been classified by Piṅgala and called *gaṇas*.

Piṅgala's *Chandaḥśāstra* as well as Lilavati and Vrittaratnakara give directions for computing the number of possible varieties and finding their places or that of any single one in a regular enumeration of them. Piṅgala is the author of *Chandaḥśāstra* (*Chandaḥsūtra*), the earliest known Sanskrit treatise on prosody.

Very less historical knowledge is available about Piṅgala, though his works are retained till date. He is identified either as the younger brother of Pāṇini (4th century BCE), or of Patañjali, the author of the Mahabhashya (2nd century BCE). His work, Chandaḥśāstra means the science of meters, is a treatise on music and can be dated back to the 2nd century BCE. The main commentaries on *Chandaḥśāstra* are

Vrittaratnakara by Kedara in the 8th century AD, *Tatparyatika* by Trivikrama in the 12th century CE, and *Mritasanjivani* by Halāyudha in 13th century CE. The complete significance of Piṅgala's work can be understood by the explanations found in these three commentaries.

Piṅgala (in *Chandaḥśāstra* 8.23) has assigned the following combinations of zero and one to represent various numbers, much in the same way as the present-day computer programming procedures. Piṅgala's system of binary numbers starts with the number one (and not zero). The numerical value is obtained by adding one to the sum of place values. In this system, the place value increases to the right, as against the modern notation in which it increases towards the left.

The procedure of the Piṅgala system is as follows:

- Divide the number by 2. If divisible write 1, otherwise write 0. If the first division yields 1 as a remainder, add 1 and divide again by 2. If fully divisible, write 1, otherwise, write 0 to the right of the first 1.
- If the first division yields 0 as the remainder that is, it is fully divisible, add 1 to the remaining number and divide by 2. If divisible, write 1, otherwise write 0 to the right of the first 0.
- This procedure is continued until 0 as the final remainder is obtained.

In the Piṅgala system, 122 can be written as 1001111.

Though this system is not the exact equivalent of today's binary system used, it is very much similar to its place value system having 20, 20, 21, 22, 22, 23, 24, 25, 26, etc used to multiply binary numbers sequence and obtain an equivalent decimal number. Reference: *Chandaḥśāstra* (8.24-25) describes the above method of obtaining the binary equivalent of any decimal number in detail.

These were used 1600 years before Westerners invented the binary system. We now use zero and one (0 and 1) in representing binary numbers, but it is not known if the concept of zero was known to Piṅgala— as a number without value and as a positional location.

Piṅgala's work also contains the Fibonacci number, called *mātrāmeru*, now known as the Gopala–Hemachandra number. Piṅgala also knew the special case of the binomial theorem for the index 2, i.e., for $(a + b)^2$, as did his Greek contemporary Euclid.

Halāyudha (10th century CE) who wrote a commentary on Piṅgala's work understood and used zero in the modern sense but by then it became commonplace.

It took several centuries before being accepted in Europe. It was Leonardo of Pisa, better known as Fibonacci, who seems to have introduced it in Europe in the 13th century. Halāyudha was himself a mathematician with no mean order. His discussion of combinatorics of poetic meters led him to a general version of the binomial theorem centuries before Newton. This too traveled east and west. Halāyudha's commentary includes a presentation of Pascal's triangle for binomial coefficients (called *meruprastāra*).

Chandaḥśāstra presents the first known description of a binary numeral system in connection with the systematic enumeration of meters with fixed patterns of short and long syllables (Short = 0, Long = 1).

As Piṅgala's system ranks binary patterns starting at one, the nth pattern corresponds to the binary representation of *n–1*, written backward. Positional use of zero dates from later centuries and would have been known to Halāyudha but not to Piṅgala.

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Shravanee Deo & Tejashree Raghothaman

Second Year BSc (Mathematics) at Fergusson College, Pune

Shravanee Deo: Studying in Second Year BSc (Mathematics) at Fergusson College, Pune, Maharashtra, India. Interested in studying applications of mathematics. Inclined to study more about the contributions of Indian Mathematicians. *Areas of interest:* Number theory, Probability theory

Tejashree Raghothaman: Studying in Second Year BSc (Mathematics) at Fergusson College, Pune, Maharashtra, India. Have a keen interest in researching minute concepts in everyday maths. Have an attraction towards and interest to know about the rich history of Ancient Indian Mathematics and its real-life applications. Also, have an affinity towards

Sanskrit in which most of Ancient Indian Mathematics is written. Reading about different mathematicians and their contributors to Indian mathematics in the past motivated me to participate in this project HoMI.
Areas of interest: Calculus, Trigonometry

Development of Mātrāvṛtta Śreṇi (Fibonacci Sequence) in India

India has contributed immensely to the field of Mathematics since its inception. Ancient Indian mathematicians have discovered and described different concepts majorly in Sanskrit. This paper intends to highlight the contributions of Indian mathematicians (Āchārya Piṅgala, Āchārya Virahānka, Āchārya Hemachandra, Nārayaṇa Pandita) and the development of one of the most celebrated concepts in Mathematics, *mātrāvṛtta śreṇi* which we now know of as the Fibonacci sequence. The first known mention of Fibonacci Numbers, prior to its discovery by Fibonacci, is found in Āchārya Piṅgala's writings. Piṅgala's rule: 'मिश्रौच'(*miśrauca*) is meant for the expansion of Mātrā Vṛttas. The definition of the mathematical relationship between the terms of the sequence was given by Āchārya Virahānka . Virahānka's rule: 'The number of variations of Mātrāvṛttas having 1, 2, 3, 4, 5, 6... mātrās as 1, 2, 3, 5, 8, 13... (or the Fibonacci Sequence).'

The formation of a number of variations of Mātrāvṛttas was first discovered and popularized by Āchārya Hemachandra. Nārāyaṇa Paṇdita defined the *Sāmāsika Pankti* (or additive sequence). Nārāyaṇa's rule: 'Putting unity twice, write their sum ahead of that. Write the sum of numbers, from the reverse order (and in) places equal to the greatest digit, ahead of that. In the absence of numbers in places equal to the greatest digit, write-ahead the sum of numbers (in available places). The number of numbers in a *Sāmāsika Pankti* happens to be '1 more than the sum'. Liber Abaci was a book published by Leonardo Bonacci, an Italian Mathematician. In this book, he gave credit to the Indian mathematicians for the development of the Fibonacci sequence.

We will describe the concepts and terms of Sanskrit which are the basic building blocks of *mātrāvṛtta śreṇi*. छन्द (Chhanda, based on *mātrās* (Guru and Laghu)), *vṛtta, śaṭ pratyayas* and *pratyayas* for *varṇa vṛttas* or syllabic metres.

Our interest lies in approaching and understanding the already-known concept of the Fibonacci sequence based on syllables which will create interest in the learners.

References: Development of Combinatorics from the *pratyayas* in Sanskrit prosody by Er. Venugopal D. Heroor

Vandita Mohta

BSc Mathematics

I am 20 years old. I am currently pursuing B.Sc (Mathematics) with an interdisciplinary concentration in Education. I finished high school at D.A.V. Girls Gopalapuram in Chennai. I can speak, read and write English and Hindi and I understand Tamil. I am familiar with Python and Latex in addition to standard software tech. Being interested in history, I have undertaken the present project on the history of games of chance under the guidance of Prof. Shailaja Sharma.

Probabilistic Rationale in Indian Dice Games

Games of chance, especially dice games, have a long history in antiquity, although they appear in the scientific literature in Europe only starting around the 15-16th centuries. We explore some dice-based plays which originated in India, in order to construct the possible mathematical rationale behind the choice of dice and point systems. We examine Pachisi and Chausar, which are forms of Ludo, played with shells and dice respectively from the standpoint of probability distributions. We notice that the dice used in the game of Chausar represent the highest variance among the alternatives over the same sample space. We present the Chausar Distribution, a symmetric discrete probability distribution, based on Chausar dice.

Keywords — Pachisi, Chausar, Indian dice, variance, games of chance

Ganesh Dharmadhikari

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Ganesh Jayantrao Dharmadhikari is our Aryabhata-HoMI Fellow working currently under the guidance of Prof. Indranath Sengupta and Prof. Michel Danino at IIT Gandhinagar. He is also a Founder of Utkalp Digital Classes an ed-tech platform where they produced 2 seasons of the multilingual podcast series ‘Science Saturdays’ in which one of the episodes discusses the story of Great Bhaskaracharya.
He completed his MS with a Mathematics major and a Physics minor from IISER Thiruvananthapuram. He is interested in various domains such as Number Theory, Algebra, Mathematical Physics, and Cryptology. He gained interest in History of Mathematics and Sciences while working at Bhaskaracharya Pratishthan, Pune.

Binary Computations and Combinatorics in Ancient India

Piṅgala is credited with the first use of the binary system, using light (*laghu*) and heavy (*guru*) syllables to describe the combinatorics of the Sanskrit meter. Piṅgala is also sometimes credited with the first use of zero, as he used the Sanskrit word *śūnya* to explicitly denote a number. *Chandaḥ Sutra* is also known as *Chandaḥśāstra* or *Piṅgala Sūtra* after its author Piṅgala. It is the earliest Hindu treatise on prosody to survive into the modern era. It is a collection of aphorisms focused mainly on the art of poetic meters and presents some mathematics mainly binary systems and Combinatorics in the service of music. We naturally discuss some interesting examples of conversion of binary to decimal and decimal to binary numbers with the help of *pratyayas* in Piṅgala’s *Chandaḥśāstra* and some poetic examples and naturally move into combinatorial methods in indian music and later discuss a few ancient Indian examples on combinatorics from *Līlavatī* and *Sangitaratnakarah*.